Position Control of a Manipulator with Passive Joints Using Dynamic Coupling

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***Abstract*—This paper describes a method of controlling the position of a manipulator composed of active and passive joints. The active joints have actuators and position sensors. The passive joints have holding brakes instead of actuators. While the brakes are released, the passive joints are indirectly controlled by the motion of the active joints using the coupling characteristics of manipulator dynamics. While the brakes are engaged, the passive joints are fixed and the active joints are controlled. The position of the manipulator is controlled by combining these two control modes. This paper describes the basic principle of the control method and the conditions that ensure the controllability of the passive joints. An algorithm for point-to-point control of the manipulator is also presented. The feasibility of the method is demonstrated by simulations for a manipulator with two degrees of freedom.**

I. Introduction

The number of degrees of freedom of a manipulator is usually equal to the number of joint actuators. In order to decrease the weight, cost, and energy consumption of a manipulator, various methods have been proposed for controlling a manipulator that has more degrees of freedom than actuators. However, these methods require special mechanisms (e.g., drive chains, drive shafts, transmission mechanisms) in addition to the basic links and joints. In this paper, a method for controlling a manipulator having more joints than actuators without using additional mechanisms is presented.

The dynamics of a manipulator has nonlinear and coupling characteristics. When each joint is controlled by a local linear feedback loop, these factors result in disturbances. The elimination of such disturbances has been one of the major problems in the control of a manipulator [1]–[5]. A design theory for a manipulator arm that has neither nonlinearity nor dynamic coupling has also been proposed [6]. However, the force of this disturbance is available to drive a joint that in itself does not have an actuator.

Vukobratović and Juričić [7] and Vukobratović and Stokić [8] proposed an algorithmic control in which the states and generalized forces of the system are partially programmed, and the unknown states and unknown generalized forces are determined by the conditions of dynamic equilibrium. This method is applied to the synthesis of biped gait [7] and biped postural stabilization [8]. In the biped system, the degrees of freedom between the foot and the ground have no actuators, and they are controlled indirectly using dynamic coupling with other powered degrees of freedom. Since dynamic equilibrium is represented as a second-order differential equation, a boundary condition is required to determine the state of the system completely. Vukobratović *et al.* used the repeatability condition of the biped gait as the boundary condition.

This paper describes a method of controlling the position of a manipulator composed of active and passive joints. The active joints have actuators and position sensors. The passive joints have holding brakes instead of actuators. While the brakes are released, the passive joints are indirectly controlled by the motion of the active joints using the coupling characteristics of manipulator dynamics. While the brakes are engaged, the passive joints are fixed and the active joints are controlled. As the passive joints can be fixed by brakes, the boundary condition concerned with the active joints can be determined arbitrarily. The total position of the manipulator is controlled by combining these two control modes. The condition that ensures controllability of the passive joints while released is obtained. An algorithm for point-to-point control of the manipulator is also presented. The feasibility of the method is demonstrated by simulations for a manipulator with two degrees of freedom.

II. Principle of Control Method

Consider a manipulator with n degrees of freedom. We assume that r ($r \geq n/2$) degrees of freedom of the manipulator are active joints with actuators and displacement sensors, which is a typical structure of manipulator joints. The $(n - r)$ degrees of freedom are passive joints that have holding brakes instead of actuators.

When the holding brakes are engaged, the active joints can be controlled without affecting the state of the passive joints. When the holding brakes are released, the passive joints can move freely and are controlled indirectly by the coupling forces generated by the motion of the active joints. The position of the manipulator is controlled by combining these two control modes.

The equations of motion of a manipulator can be written as follows

$$M(q)\ddot{q} + b(q, \dot{q}) = u \tag{1}$$

where

$$b(q, \dot{q}) = h(q, \dot{q}) + \Gamma\dot{q} + g(q)$$

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and

$q \in R^n$	are joint displacements,
$u \in R^n$	are generalized forces,
$g(q) \in R^n$	are gravitational forces,
$h(q, \dot{q}) \in R^n$	are Coriolis and centrifugal forces,
$M(q) \in R^{n\times n}$	is the inertia matrix, and
$\Gamma \in R^{n\times n}$	is the viscosity friction matrix.

The displacements of r joints, including all the $(n-r)$ passive joints, are selected from the elements of q and set as a vector $\psi \in R^r$ (since $r \geq n/2$, $r \geq n-r$). When $r > n-r$, ψ is composed of the displacements of the $(n-r)$ passive joints $\psi_{pas} \in R^{n-r}$ and those of the $(2r-n)$ active joints $\psi_{act} \in R^{2r-n}$. When $r = n-r$, all the joints represented by ψ are passive. In addition, all the remaining $(n-r)$ joints are active, and their displacements are represented as $\phi \in R^{n-r}$. The generalized forces of the r active joints are expressed as $\tau \in R^r$. When the passive joints are free, the generalized forces of the passive joints are equal to zero. The elements of q and u are then rearranged as

$$q = \begin{bmatrix}\phi\\ \psi\end{bmatrix}\begin{matrix}n-r\\ r\end{matrix} = \begin{bmatrix}\phi\\ \psi_{act}\\ \psi_{pas}\end{bmatrix}\begin{matrix}n-r\\ 2r-n\\ n-r\end{matrix}$$

$$u = \begin{bmatrix}\tau\\ 0\end{bmatrix}\begin{matrix}r\\ n-r\end{matrix}. \tag{2}$$

Accordingly, $M(q)$ and $b(q, \dot{q})$ are also rearranged and partitioned as follows:

$$M(q) = \begin{bmatrix}M_{11}(q) & M_{12}(q)\\ M_{21}(q) & M_{22}(q)\end{bmatrix}\begin{matrix}r\\ n-r\end{matrix}$$
$$\qquad\quad n-r \qquad\quad r$$

$$b(q, \dot{q}) = \begin{bmatrix}b_1(q, \dot{q})\\ b_2(q, \dot{q})\end{bmatrix}\begin{matrix}r\\ n-r\end{matrix}. \tag{3}$$

When (2) and (3) are substituted for (1), we obtain

$$M_{11}\ddot{\phi} + M_{12}\ddot{\psi} + b_1 = \tau \tag{4a}$$

$$M_{21}\ddot{\phi} + M_{22}\ddot{\psi} + b_2 = 0. \tag{4b}$$

The elements of M are functions of q. The elements of b are functions of q and $\dot{q}$. M_{11}, M_{12}, M_{21}, M_{22}, b_1, and b_2 can be calculated from the dynamic model of the manipulator if the measured value of joint displacement and velocity at each joint is substituted in q and $\dot{q}$ of (3). Furthermore, when desired values $\ddot{\psi}_d$ are assigned to the accelerations $\ddot{\psi}$, (4b) is considered a linear equation with regard to $\ddot{\phi}$. The coefficient matrix M_{21} corresponds to the dynamic coupling between the accelerations of ϕ and the generalized forces of the passive joints, and it depends upon the structure and mass distribution of the manipulator. If M_{21} is nonsingular, (4b) can be solved uniquely for $\ddot{\phi}$ as

$$\ddot{\phi} = -M_{21}^{-1}M_{22}\ddot{\psi}_d - M_{21}^{-1}b_2. \tag{5}$$

When (5) is substituted in (4a), we obtain

$$\tau = \left(M_{12} - M_{11}M_{21}^{-1}M_{22}\right)\ddot{\psi}_d + b_1 - M_{11}M_{21}^{-1}b_2. \tag{6}$$

If we apply these generalized forces τ to the active joints, the resulting accelerations will be $\ddot{\phi}$ and $\ddot{\psi}_d$.

In other words, while the passive joints are free, the accelerations $\ddot{\psi}$ of r joints, including the $(n-r)$ passive joints ψ_{pas}, can be arbitrarily determined by applying the generalized forces τ to the r active joints. The accelerations $\ddot{\phi}$ of the remaining $(n-r)$ active joints are determined by $\ddot{\psi}_d$ and cannot be adjusted arbitrarily. However, in addition to the accelerations, the initial displacements and velocities are required to prescribe the motion completely. The initial displacements of the passive joints are the displacements at the moment the brakes are released. The initial velocities of the passive joints are zero. The initial displacements and velocities of the active joints are determined arbitrarily by controlling the active joints with the passive joints fixed.

While the passive joints are fixed, their velocities and accelerations are zero. The displacements, velocities, and accelerations of the active joints can then be controlled without affecting the passive joints.

By organizing the motion pattern using a combination of these two modes, the total position of the manipulator is controlled. In Section IV, an example of the organization of this pattern is described.

III. Controllability and Output Controllability

In this section, we present the theoretical basis of the proposed method from the standpoint of linear system theory [9]. First, we investigate the controllability of a manipulator system with passive joints. The system is uncontrollable (in the sense of linear system theory) if no gravity and no friction act on the passive joints. In such a case, simultaneous positioning of all the joints is impossible. Therefore, the holding brakes of the passive joints are necessary to ensure the positioning capability of the manipulator.

Second, we investigate the output controllability of the system. The output is composed of displacements and velocities of some of the joints. For output controllability, the number of joints we assign displacements and velocities (output joints) must be less than or equal to the number of active joints. Including the released passive joints, we can simultaneously control as many joints as the number of active joints. Therefore, the number of passive joints should be less than or equal to the number of active joints.

Finally, we show the relationship between the linear approximation and analytical expression by proving that the conditions of output controllability are equivalent to the conditions necessary to solve the generalized forces of the active joints (i.e., M_{21} is nonsingular).

A. Linear Approximation Model

At first, the equations of motion of the manipulator are linearized to obtain a state variable representation. Since the inertia matrix $M(q)$ is generally nonsingular, (1) is transformed as

$$\ddot{q} - M(q)^{-1}u + M(q)^{-1}b(q, \dot{q}) = 0. \tag{7}$$

In order to ensure equilibrium while the passive joints are free, it is assumed that gravitational forces do not act on the passive joints and that the friction of the passive joints can be ignored. When (7) is linearized in the neighborhood of the equilibrium point, the system can be represented by the following state equation:

$$\dot{x} = Ax + B\tau$$

$$A = \begin{bmatrix} 0 & 0 & I_{n-r} & 0 \\ 0 & 0 & 0 & I_r \\ -N_{11}\dfrac{\partial b_1}{\partial \phi} & -N_{11}\dfrac{\partial b_1}{\partial \psi} & -N_{11}\dfrac{\partial b_1}{\partial \dot{\phi}} & -N_{11}\dfrac{\partial b_1}{\partial \dot{\psi}} \\ -N_{21}\dfrac{\partial b_1}{\partial \phi} & -N_{21}\dfrac{\partial b_1}{\partial \psi} & -N_{21}\dfrac{\partial b_1}{\partial \dot{\phi}} & -N_{21}\dfrac{\partial b_1}{\partial \dot{\psi}} \end{bmatrix} \begin{matrix} n-r \\ r \\ n-r \\ r \end{matrix}$$
$$\qquad\qquad n-r \qquad\quad r \qquad\quad n-r \qquad\quad r$$

$$B = \begin{bmatrix} 0 \\ 0 \\ N_{11} \\ N_{21} \end{bmatrix} \begin{matrix} n-r \\ r \\ n-r \\ r \end{matrix}$$
$$r$$

$$x = \begin{bmatrix} \delta\phi \\ \delta\psi \\ \delta\dot{\phi} \\ \delta\dot{\psi} \end{bmatrix} \begin{matrix} n-r \\ r \\ n-r \\ r \end{matrix}$$
$$r.$$

where

$$M(q)^{-1} = \begin{bmatrix} N_{11}(q) & N_{12}(q) \\ N_{21}(q) & N_{22}(q) \end{bmatrix} \begin{matrix} n-r \\ r \end{matrix}. \tag{9}$$
$$r \qquad n-r$$

Next, we make the variable transformation $x = Tz$ using T

$$\dot{z} = \bar{A}z + \bar{B}\tau \tag{12}$$

where

$$\bar{A} = T^{-1}AT = \begin{bmatrix} -\dfrac{\partial b_1}{\partial \dot{\phi}}N_{11} - \dfrac{\partial b_1}{\partial \dot{\psi}}N_{21} & -\dfrac{\partial b_1}{\partial \phi}N_{11} - \dfrac{\partial b_1}{\partial \psi}N_{21} & -\dfrac{\partial b_1}{\partial \dot{\phi}}N_{12} - \dfrac{\partial b_1}{\partial \dot{\psi}}N_{22} & -\dfrac{\partial b_1}{\partial \phi}N_{12} - \dfrac{\partial b_1}{\partial \psi}N_{22} \\ I_r & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & I_{n-r} & 0 \end{bmatrix}$$

$$\bar{B} = T^{-1}B = \begin{bmatrix} I_r \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$z = T^{-1}x = \begin{bmatrix} M_{11}\delta\dot{\phi} + M_{12}\delta\dot{\psi} \\ M_{11}\delta\phi + M_{12}\delta\psi \\ M_{21}\delta\dot{\phi} + M_{22}\delta\dot{\psi} \\ M_{21}\delta\phi + M_{22}\delta\psi \end{bmatrix}.$$

The state variables $\delta\phi$, $\delta\psi$, $\delta\dot{\phi}$, $\delta\dot{\psi}$ consist of the deviations of the joint displacements and velocities from the equilibrium point. The input is the generalized forces τ of the active joints (in this case, the number of inputs r and choice of ϕ, ψ is arbitrary, so it is not necessary for ψ to include all the passive joints).

B. Controllability

Here, the $2n \times 2n$ matrix T is introduced as

$$T = \begin{bmatrix} 0 & N_{11} & 0 & N_{12} \\ 0 & N_{21} & 0 & N_{22} \\ N_{11} & 0 & N_{12} & 0 \\ N_{21} & 0 & N_{22} & 0 \end{bmatrix} \tag{10}$$

and T is nonsingular. T^{-1} is given by

$$T^{-1} = \begin{bmatrix} 0 & 0 & M_{11} & M_{12} \\ M_{11} & M_{12} & 0 & 0 \\ 0 & 0 & M_{21} & M_{22} \\ M_{21} & M_{22} & 0 & 0 \end{bmatrix}. \tag{11}$$

When the controllability matrix $\bar{V}$ is calculated to investigate the controllability of system (12)

$$\bar{V} = [\bar{B} \;\; \bar{A}\bar{B} \;\; \cdots \;\; \bar{A}^{n-1}\bar{B}]$$
$$= \begin{bmatrix} I_r & -\dfrac{\partial b_1}{\partial \dot{\phi}}N_{11} - \dfrac{\partial b_1}{\partial \dot{\psi}}N_{21} & * & \cdots \\ 0 & I_r & ** & \cdots \\ 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots \end{bmatrix}. \tag{13}$$

Since rank $[\overline{V}] = 2r < 2n$, system (12) is uncontrollable. Hence, system (8), which is equivalent to (12), is also uncontrollable. In other words, when the displacements and velocities of n joints are simultaneously controlled with the generalized forces of r ($< n$) active joints, there exists a domain that the manipulator cannot reach by an input even in the neighborhood of the equilibrium point.

C. Output Controllability

Here, the output of the system (12) is taken as

$$y = \overline{C}z$$

$$y = \begin{bmatrix} \delta\psi \\ \delta\dot{\psi} \end{bmatrix}$$

$$\overline{C} = \underset{\quad r \qquad\quad r \qquad\quad n-r \qquad n-r}{\begin{bmatrix} 0 & N_{21} & 0 & N_{22} \\ N_{21} & 0 & N_{22} & 0 \end{bmatrix}}\begin{matrix} r \\ r \end{matrix}. \tag{14}$$

The displacements and velocities of r joints are the output. If the system is output controllable, the existence of an input τ that transfers the output $\delta\psi$, $\delta\dot{\psi}$ from an arbitrary value to zero is guaranteed. The complete condition for output controllability of systems (12) and (14) is that the rank of matrix $\overline{N} = [\overline{C}\overline{B} \ \ \overline{C}\overline{A}\overline{B} \ \ \overline{C}\overline{A}^2 \ \overline{B} \cdots \overline{C}\overline{A}^{n-1} \ \overline{B}]$ equals $2r$.

$$\overline{N} = \overline{C}\,\overline{V}$$

$$= \begin{bmatrix} 0 & N_{21} \\ N_{21} & 0 \end{bmatrix} \begin{bmatrix} I_r & -\dfrac{\partial b_1}{\partial \phi}N_{11} - \dfrac{\partial b_1}{\partial \psi}N_{21} & * & \cdots \\ 0 & I_r & ** & \cdots \end{bmatrix}. \tag{15}$$

From (15), the condition of output controllability is equivalent to the nonsingularity of N_{21}. Hence, if N_{21} is nonsingular, it is possible to set ψ and $\dot{\psi}$ at desired values and to perform local positioning in the neighborhood of the equilibrium point.

Recall that r is the number of active joints. If $r \geq n - r$, we can select the r joints represented by ψ so as to include the $(n - r)$ passive joints. In such a case, all the passive joints can be positioned with the generalized forces of the active joints. On the other hand, the displacements and velocities of joints greater than r are not output controllable irrespective of the output matrix $\overline{C}$ since the rank of matrix $\overline{N}$ in (15) is less than or equal to $2r$. Hence, at least $(n - 2r)$ passive joints cannot be controlled if $r < n - r$.

Furthermore, it can be proved that this condition agrees with the condition that M_{21} must be nonsingular, and therefore (4b) has a solution. This means that if and only if the accelerations of r joints, including $(n - r)$ passive joints, can be arbitrarily determined by the generalized forces of r active joints without linear approximation, systems (12) and (14) become output controllable.

Proposition: When the n dimensional square matrix

$$M = \underset{\quad n-r \qquad r}{\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}}\begin{matrix} r \\ n-r \end{matrix} \tag{a}$$

is nonsingular and the inverse matrix of M is represented as

$$M^{-1} = \underset{\quad n \qquad n-r}{\begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix}}\begin{matrix} n-r \\ r \end{matrix} \tag{b}$$

the complete condition that M_{21} is nonsingular is that N_{21} is nonsingular.

Proof: First, the sufficient condition is verified. Since N_{21} is nonsingular, when

$$Q = \underset{\quad n-r \qquad r}{\begin{bmatrix} I_{n-r} & -N_{11}N_{21}^{-1} \\ 0 & I_r \end{bmatrix}}\begin{matrix} n-r \\ r \end{matrix} \tag{c}$$

is postmultipled using both sides of (b)

$$QM^{-1} = \underset{\quad r \qquad\qquad n-r}{\begin{bmatrix} 0 & N_{12} - N_{11}N_{21}^{-1}N_{22} \\ N_{21} & N_{22} \end{bmatrix}}\begin{matrix} n-r \\ r \end{matrix}. \tag{d}$$

Since both Q and M are nonsingular, QM^{-1} is also nonsingular. Accordingly, the vectors from the first to the $(n - r)$th row of QM^{-1} are linearly independent and $N_{12} - N_{11}N_{21}^{-1}N_{22}$ $(= N*)$ is nonsingular. Further, if

$$P = \underset{\quad n-r \qquad r}{\begin{bmatrix} I_{n-r} & 0 \\ -N_{22}N_*^{-1} & I_r \end{bmatrix}}\begin{matrix} n-r \\ r \end{matrix} \tag{e}$$

is postmultiplied using both sides of (d)

$$PQM^{-1} = \underset{\quad r \qquad n-r}{\begin{bmatrix} 0 & N_* \\ N_{21} & 0 \end{bmatrix}}\begin{matrix} n-r \\ r \end{matrix}. \tag{f}$$

Next, we postmultiply (f) by M to obtain

$$PQ = PQM^{-1}M$$
$$= \begin{bmatrix} N_*M_{21} & N_*M_{22} \\ N_{21}M_{11} & N_{21}M_{12} \end{bmatrix}. \tag{g}$$

Since

$$PQ = \begin{bmatrix} I_{n-r} & -N_{11}N_{21}^{-1} \\ -N_{22}N_*^{-1} & I_r + N_{22}N_*^{-1}N_{11}N_{21}^{-1} \end{bmatrix} \tag{h}$$

from (c) and (e), $M_{21} = N_*^{-1}$. Accordingly, M_{21} is nonsingular. With respect to the necessary condition, the exact same proof can be made when M and N are exchanged. ■

IV. Control Algorithm

In controlling the position of a manipulator, several control algorithms can be devised according to the way in which the two control modes are combined. As an example, point-to-point (PTP) control is considered here.

Since the passive joints are controlled with the brakes released (brakes-OFF) and the active joints are controlled with the brakes engaged (brakes-ON), the control mode should be

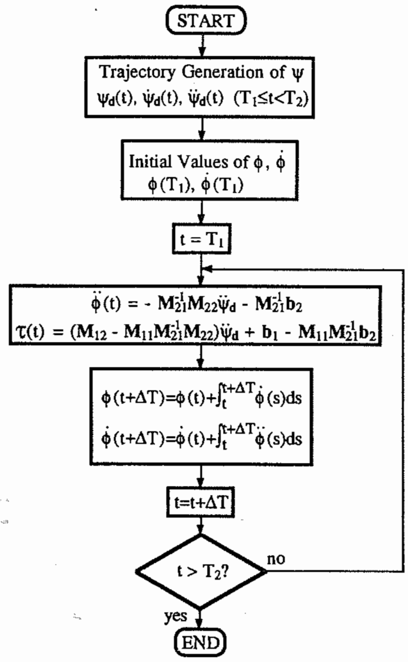

Fig. 1. Control algorithm in brake-OFF period.

changed at least once so all the joints of the manipulator can reach the desired position. It is considered difficult to control both the passive and active joints simultaneously to reach the desired position precisely while the passive joints are free. In the simplest control algorithm sequence, therefore, first the passive joints are positioned while the passive joints are free, then the remaining active joints are positioned while the passive joints are fixed.

PTP control is possible with this sequence alone, but to make it even easier, a brakes-ON period is added before the brakes-OFF period. This brakes-ON period provides kinetic energy to the link mechanism through the active joints so that the passive joints can move easily and thus unnecessary motion of the active joints is reduced. Consequently, the period of positioning is divided into the following three phases:

Phase I: ψ joints are fixed ($T_0 \le t < T_1$, brakes ON)
Phase II: ψ joints are free ($T_1 \le t < T_2$, brakes OFF)
Phase III: ψ joints are fixed ($T_2 \le t < T_3$, brakes ON)

In phase II, r joints, including all the passive joints, are controlled along a desired trajectory. In phases I and III, the remaining $(n - r)$ active joints are controlled from the initial position to the final position.

First, the trajectory and the generalized forces in phase II are calculated off-line as follows (Fig. 1).

Step 1: Desired displacements, velocities and accelerations $\psi_d(t)$, $\dot{\psi}_d(t)$, $\ddot{\psi}_d(t)$ $(T_1 \le t < T_2)$ of r joints, including all the passive joints, are prescribed. (In addition, $\dot{\psi}_d(T_1) = \dot{\psi}_d(T_2) = 0$ is a boundary condition because at the moment of brake switching, the passive joints must be at rest.)

Step 2: The initial displacements $\phi(T_1)$ and velocities $\dot{\phi}(T_1)$ of the remaining $(n - r)$ joints are assigned.

Step 3: Equations (5) and (6) are used to determine $\tau(T_1)$ and $\ddot{\phi}(T_1)$.

Step 4: $\phi(T_1 + \Delta T)$ and $\dot{\phi}(T_1 + \Delta T)$ are determined by numerical integration of $\ddot{\phi}(t)$. Here, ΔT is the sampling interval.

Step 5: Steps 3 and 4 are repeated, and the generalized forces τ, accelerations $\ddot{q}$, velocities $\dot{q}$, and displacements q are determined for the rest of phase II.

The results of these off-line calculations are used for feedforward compensation. Moreover, the boundary values of ϕ and $\dot{\phi}$ between the phases I, III, and II are determined using this result.

When the control is executed in real time, the following feedback control is applied:

$$\ddot{\psi}_d' = \ddot{\psi}_d + K_v(\dot{\psi}_d - \dot{\psi}) + K_p(\psi_d - \psi) + K_i \int (\psi_d - \psi)\, dt,$$
$$(K_v,\ K_p,\ K_i\text{: diagonal gain matrix}). \tag{16}$$

Here, ψ_d, $\dot{\psi}_d$, and $\ddot{\psi}_d$ are the desired values of displacements, velocities, and accelerations given in step 1 and ψ and $\dot{\psi}$ are the measured values of displacements and velocities. Accelerations $\ddot{\psi}_d'$ obtained in (16) are substituted in $\ddot{\psi}_d$ of (6) and the generalized forces τ are determined. In this case, the actual values for ϕ and $\dot{\phi}$ can be obtained by measurement, and numerical integration is not required. From (4a), (5), (6), and (16)

$$(\ddot{\psi}_d - \ddot{\psi}) + K_v(\dot{\psi}_d - \dot{\psi}) + K_p(\psi_d - \psi) + K_i \int (\psi_d - \psi)\, dt = 0. \tag{17}$$

Therefore, $\psi_d - \psi$ is guraranteed to converge to zero if K_v, K_p, and K_i are correctly chosen.

In phases I and III, the control of the joint angles ϕ is similar to an ordinary manipulator. In phase I, ϕ is controlled from the initial position $\phi(T_0)$ to $\phi(T_1)$ and $\dot{\phi}$ is controlled from $\dot{\phi}(T_0) = 0$ to $\dot{\phi}(T_1)$. In phase III, ϕ is controlled from $\phi(T_2)$ to the final position $\phi(T_3)$ and $\dot{\phi}$ is controlled from $\dot{\phi}(T_2)$ to $\dot{\phi}(T_3) = 0$. Using the algorithm discussed above, the position of the manipulator can be controlled between any two points.

V. SIMULATION

In order to confirm the feasibility of the proposed method, the control of a two-degrees-of-freedom manipulator (Fig. 2) is considered. The mass distributions of links 1 and 2 are assumed to be uniform. The required actuator torque, the positioning time, etc. are then evaluated. Specifically, (5) and (6) are expressed as follows:

$$\ddot{\phi} = -\frac{M_{22}\ddot{\psi} + D_{211}(\psi)\dot{\phi}^2}{M_{21}(\psi)} \tag{18}$$

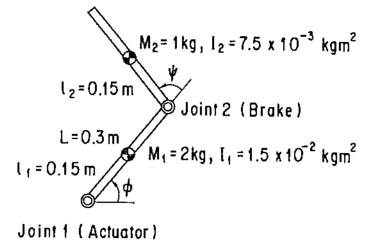

Fig. 2. Two-degrees-of-freedom manipulator.

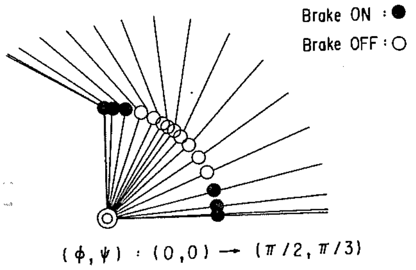

Fig. 3. Simulated motion of the manipulator.

TABLE I
SIMULATION RESULTS

Positioning Time	$T_3 - T_0 \le 1.78$ s
Actuator Torque	$\lvert \tau_\phi \rvert \le 2.3$ Nm
Actuator Velocity	$\lvert \dot\phi \rvert \le 2.9$ rad/s
Actuator Acceleration	$\lvert \ddot\phi \rvert \le 12.6$ rad/s^2
Brake Torque	$\lvert \tau_\psi \rvert \le 0.96$ Nm

$$\tau = \left(M_{12}(\psi) - \frac{M_{12}(\psi) M_{22}}{M_{21}(\psi)} \right) \ddot\psi + D_{112}(\psi)\dot\phi\dot\psi$$

$$+ D_{122}(\psi)\dot\psi^2 - \frac{M_{11}(\psi) D_{211}(\psi)}{M_{21}(\psi)} \dot\phi^2.$$

The trajectory of ψ in phase II is assigned arbitrarily. The acceleration/deceleration curve is chosen to follow a sinusoidal function in order to reduce the required actuator torque. The maximum value of $\lvert \ddot\psi \rvert$ is assigned so that the actuator torque is limited to a certain value, T_2-T_1 is obtained from the values of $\psi(T_1)$ and $\psi(T_2)$.

The initial values of ϕ and $\dot\phi$ ($\phi(T_1)$, $\dot\phi(T_1)$) can be arbitrarily selected. The trajectory of ϕ is determined by steps 2–5 in Section IV, and thereafter $\phi(T_2)$ and $\dot\phi(T_2)$ are calculated. The acceleration/deceleration of ϕ along phases I and III also follows a sinusoidal function.

An example of PTP control is illustrated using a stick diagram of the two-link manipulator (Fig. 3). Since (18) and (19) have solutions only when $M_{21} \neq 0$, control of the manipulator is possible within the domain of $-2.30 < \psi < 2.30$. The maximum positioning time, maximum torque, and so on are evaluated by varying the values of $\phi(T_3)$, $\psi(T_0)$ and $\psi(T_3)$ by $\pi/6$ steps within the following domain (Table I).

$$\phi(T_0) = 0, -\frac{\pi}{2} \le \psi(T_0) \le \frac{\pi}{2},$$

$$0 \le \phi(T_3) \le \frac{\pi}{2}, -\frac{\pi}{2} \le \psi(T_3) \le \frac{\pi}{2}.$$

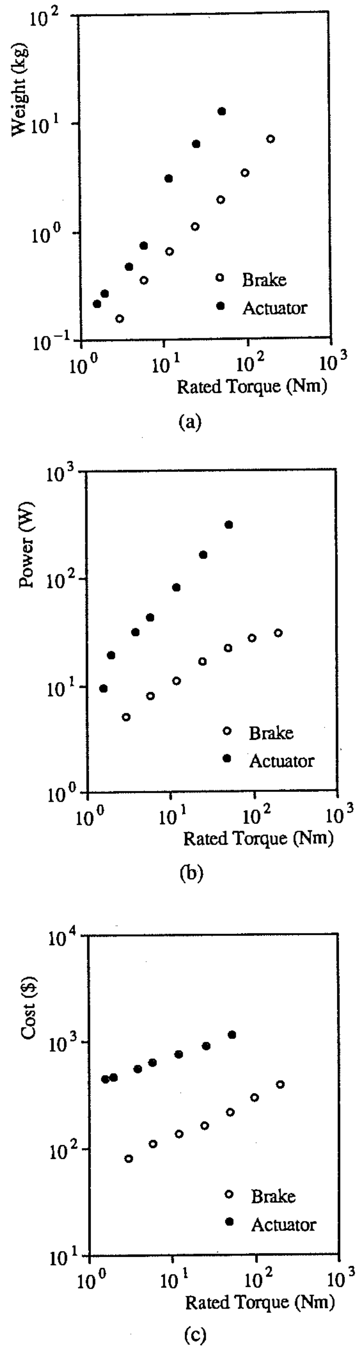

Fig. 4. Comparison between actuators and brakes. Actuator: dc servo motor with harmonic-drive gears. Brake: electromagnetic brake with a friction disk. (a) Weight. (b) Energy consumption. (c) Cost.

In this case, the maximum values of $\lvert \ddot\phi \rvert$ in phases I and III and $\lvert \ddot\psi \rvert$ in phase II are 4π rad/s^2. Within this domain, positioning between any two points is completed within 1.8 s, and the performance required of the actuator and the brake can be satisfied with commercially available components.

Furthermore, the coefficients that depend upon the mass and length of the link in (18) are cancelled out. Therefore, so long as the configuration and mass distribution of the manipulator are geometrically similar, the positioning time $T_3 - T_0$ yields the same maximum values with respect to the same maximum value of acceleration $\lvert \ddot\phi \rvert$ and $\lvert \ddot\psi \rvert$, irrespective of the link mass or length.

VI. Conclusion

In this paper, a dynamic coupling method has been proposed for controlling the position of a manipulator with passive joints. The paper also described the basic principles and the conditions under which control is possible. The feasibility of the method is demonstrated by simulations for a manipulator with two degrees of freedom. In addition, fundamental experiments are now being carried out with an experimental manipulator.

Since the proposed method depends on a dynamic model of a manipulator, this method is more effective when applied in combination with dynamic modeling and identification methods [10].

We examined the controllability of the manipulator using the nonsingularity condition of the inertia matrix. However, it is preferable to represent the degree of controllability with quantitative criteria like output-controllability gain [9].

When some joint actuators of a manipulator are exchanged for holding brakes with the proposed method, we can build a light-weight, energy-saving, low-cost manipulator. As an example, in Fig. 4 we compare the weight, energy consumption, and cost of brakes (electromagnetic type with a friction disk) with those of actuators (dc servo motor with harmonic-drive reduction gears). The parameters of brakes are much smaller than those of the actuators in all respects. We can take advantage of these merits by applying them to simple assembly robots, control of redundant manipulators, etc. Space applications (e.g., space manipulators, expansion of space structures) may also be effective because there is no gravity influence.

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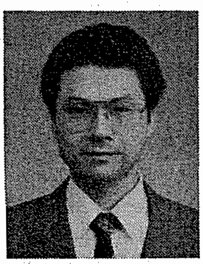

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