POSITION CONTROL OF A MANIPULATOR WITH PASSIVE JOINTS USING DYNAMIC COUPLING

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ABSTRACT

This paper describes a method of controlling the position of a manipulator with passive joints. The passive joints have holding brakes instead of actuators. While the brakes are released, the passive joints are indirectly controlled by the motion of the active joints using the coupling characteristics of manipulator dynamics. While the brakes are engaged, the active joints are controlled. This paper describes the basic principle and the conditions that ensure the controllability of the passive joints. An algorithm for PTP control is also presented. The feasibility of the method is demonstrated by computer simulation.

1. INTRODUCTION

The number of degrees of freedom that a manipulator possesses is commonly equal to the number of joint actuators. In order to decrease weight,cost and energy consumption of a manipulator, various approaches have been proposed for controlling a manipulator which has more degrees of freedom than actuators. However, they require special mechanisms (e.g. drive chains, drive shafts, transmission mechanisms) in addition to the basic links and joints. In this paper, an approach to control a manipulator which has more joints than actuators without additional mechanisms is presented.

The dynamics of a manipulator has non-linear and coupling characteristics. When each joint is controlled by a local linear feedback loop, these factors result in disturbance. The elimination of such disturbances has been one of the major problems in the control of a manipulator [1,2,3,4]. A design theory of a manipulator arm which has neither non-linearity nor dynamic coupling has also been proposed [5]. However, the force of this disturbance is available to drive a joint which in itself does not have an actuator.

Vukobratović [6,7] proposed algorithmic control, in which the state and generalized force of the system are partially programmed, and unknown states and unknown generalized forces are determined by the conditions of dynamic equilibrium. This method is applied to synthesis of biped gait [6] and biped postural stabilization [7]. In the biped system, the degrees of freedom between foot and ground have no actuators, and they are controlled indirectly using dynamic coupling with other powered degrees of freedom. Since the dynamic equilibrium is represented as a second-order differential equation, a boundary condition is required to determine the state of the system completely. Vukobratović used the repeatability condition of the biped gait as the boundary condition.

This paper describes a method of controlling the position of a manipulator which is composed of active and passive joints. The active joints have actuators and position sensors. The passive joints have holding brakes instead of actuators. While the brakes are released, the passive joints are indirectly controlled by the motion of the active joints using the coupling characteristics of manipulator dynamics. While the brakes are engaged, the passive joints are fixed and the active joints are controlled. As the passive joints can be fixed by brakes, the boundary condition concerned with the active joints can be determined arbitrarily. The total position of the manipulator is controlled by combining these two control modes. The condition that ensures controllability of the passive joints while released is obtained. An algorithm for point to point control of the manipulator is also presented. The feasibility of the method is demonstrated by simulation experiments of a manipulator with two degrees of freedom.

2. PRINCIPLE OF CONTROL METHOD

Consider a manipulator with n degrees of freedom. We assume that $r(r \geqq n/2)$ degrees of freedom of the manipulator are active joints with actuators and displacement sensors, which is a common structure of manipulator joints. The (n-r) degrees of freedom are passive joints which have holding brakes instead of actuators.

When the holding brakes are engaged, the active joints can be controlled without affecting the state of the passive joints. When the holding brakes are released, the passive joints can move freely and are controlled indirectly by the coupling force generated by the motion of the active joints. The position of the manipulator is controlled by combining these two control modes.

The equation of motion of a manipulator can be written as

$$\mathbf{M}(q)\ddot{q} + b(q,\dot{q}) = \mathbf{u} \qquad (1)$$

where,

$$b(q,\dot{q}) = h(q,\dot{q}) + \Gamma\dot{q} + g(q)$$

$q \in R^n$	: joint displacement	$u \in R^n$	: generalized force
$g(q) \in R^n$	: gravity	$h(q,\dot{q}) \in R^n$	: Coriolis'/centrifugal force
$\mathbf{M}(q) \in R^{n \times n}$	: inertia matrix	$\Gamma \in R^{n \times n}$	: viscosity friction matrix

The displacements of r joints, including all the (n-r) passive joints, are selected from the elements of q and set as a vector $\psi \in R^r$ (since $r \geqq n/2$, $r \geqq n-r$). All the remaining (n-r) joints are active and their displacements are represented as $\phi \in R^{n-r}$. Moreover, the generalized force of the r active joints is expressed as $\tau \in R^r$. When the passive joints are free, the generalized force of the passive joints is equal to zero.

The elements of q and u are rearranged as

$$q = \begin{bmatrix} \phi \\ \psi \end{bmatrix} \begin{matrix} {\scriptstyle n-r} \\ {\scriptstyle r} \end{matrix} \qquad u = \begin{bmatrix} \tau \\ 0 \end{bmatrix} \begin{matrix} {\scriptstyle r} \\ {\scriptstyle n-r} \end{matrix} \qquad (2)$$

Accordingly, $\mathbf{M}(q)$ and $b(q,\dot{q})$ are also rearranged and $\mathbf{M}(q)$ is partitioned as follows.

$$\mathbf{M}(q) = \begin{array}{c} \begin{bmatrix} \mathbf{M}_{11}(q) & \mathbf{M}_{12}(q) \\ \mathbf{M}_{21}(q) & \mathbf{M}_{22}(q) \end{bmatrix} \begin{matrix} {\scriptstyle r} \\ {\scriptstyle n-r} \end{matrix} \\ \begin{matrix} {\scriptstyle n-r} \qquad\qquad {\scriptstyle r} \end{matrix} \end{array} \qquad (3)$$

When (2) and (3) are substituted for (1),

$$\mathbf{M}_{11}\ddot{\phi} + \mathbf{M}_{12}\ddot{\psi} + [I_r \quad 0]b(q,\dot{q}) - \tau = 0 \qquad (4a)$$
$$\mathbf{M}_{21}\ddot{\phi} + \mathbf{M}_{22}\ddot{\psi} + [0 \quad I_{n-r}]b(q,\dot{q}) = 0 \qquad (4b)$$

$\mathbf{M}_{11}$, $\mathbf{M}_{12}$, $\mathbf{M}_{21}$, $\mathbf{M}_{22}$ and $b(q,\dot{q})$ can be determined if the measured value of joint displacement and velocity at each joint is substituted in $q,\dot{q}$ of (4a) and (4b). Furthermore, when a desired value ($=\ddot{\psi}_d$) is assigned to the acceleration $\ddot{\psi}$, (4b) is considered as a linear equation with regard to $\ddot{\phi}$. The coefficient matrix $\mathbf{M}_{21}$ corresponds to the dynamic coupling between ϕ and ψ, and it depends upon the structure and mass distribution of the manipulator. If $\mathbf{M}_{21}$ is non-singular, (4b) can be solved uniquely

$$\ddot{\phi} = -\mathbf{M}_{21}^{-1}\mathbf{M}_{22}\ddot{\psi}_d - [0 \quad \mathbf{M}_{21}^{-1}]b(q,\dot{q}) \qquad (5)$$

When (5) is substituted in (4a),

$$\tau = (\mathbf{M}_{12} - \mathbf{M}_{11}\mathbf{M}_{21}^{-1}\mathbf{M}_{22})\ddot{\psi}_d + [I_r \quad -\mathbf{M}_{11}\mathbf{M}_{21}^{-1}]b(q,\dot{q}) \qquad (6)$$

If the generalized force τ thus derived actuates the active joints, acceleration $\ddot{\phi}$, $\ddot{\psi}_d$ arise.

In other words, while the passive joints are free, the acceleration $\ddot{\psi}$ of r joints, including the (n-r) passive joints, can be arbitrarily determined by the generalized force of the r active joints. The acceleration $\ddot{\phi}$ of the remaining (n-r) active joints is determined by $\ddot{\psi}_d$ and cannot be adjusted arbitrarily. However, in addition to acceleration, initial conditions of displacement and velocity are required to prescribe motion completely. Initial displacement of the passive joints is that of the moment when the brakes are released. Initial velocity of the passive joints is zero. Initial displacement and velocity of the active joints is determined arbitrarily by controlling them while the passive joints are fixed.

While the passive joints are fixed, their velocity and acceleration are zero. The displacement, velocity and acceleration of the active joints can be controlled without affecting the passive joints. By organizing the pattern of the motion by a combination of these two modes, the total position of a manipulator is controlled.

3. CONTROLLABILITY AND OUTPUT-CONTROLLABILITY

In this chapter, the basis of the proposed method is given from the standpoint of linear system theory[8]. First of all, we show that the system of a manipulator with passive joints is generally uncontrollable (in the sense of linear system theory), and simultaneous positioning of all the joints is impossible. Therefore, holding brakes of passive joints are necessary.

Secondly, we propose that the joints of the same number as the active joints, including all the passive joints, are controlled while the passive joints are free. Therefore the number of the passive joints should be less than or equal to the number of the active joints. We explain it by showing that the system is output-controllable if output of the system is assigned to displacement and velocity of the joints of the same number as the active joints.

3.1 Linear Approximation Model

At first, the equation of motion of the manipulator is linearized to obtain state variable representation. Since the inertia matrix $\mathbf{M}(\mathbf{q})$ is generally non-singular, (1) is transformed as

$$\ddot{\mathbf{q}} - \mathbf{M}(\mathbf{q})^{-1}\mathbf{u} + \mathbf{M}(\mathbf{q})^{-1}\mathbf{b}(\mathbf{q},\dot{\mathbf{q}}) = 0 \qquad (7)$$

It is assumed that gravitational force and friction of the joints can be ignored. When (7) is linearized in the neighborhood of the equilibrium point, the system can be represented by the following state equation.

$$\dot{x} = Ax + B\tau \qquad (8)$$

$$A = \begin{array}{cc} \begin{bmatrix} 0 & I_n \\ 0 & 0 \end{bmatrix} & \begin{array}{c} n \\ n \end{array} \\ \begin{array}{cc} n & n \end{array} & \end{array} \qquad B = \begin{array}{cc} \begin{bmatrix} 0 \\ 0 \\ N_{11} \\ N_{21} \end{bmatrix} & \begin{array}{c} n-r \\ r \\ n-r \\ r \end{array} \\ r & \end{array} \qquad x = \begin{array}{cc} \begin{bmatrix} \delta\phi \\ \delta\psi \\ \delta\dot{\phi} \\ \delta\dot{\psi} \end{bmatrix} & \begin{array}{c} n-r \\ r \\ n-r \\ r \end{array} \end{array}$$

where,

$$\mathbf{M}(\mathbf{q})^{-1} = \begin{array}{cc} \begin{bmatrix} N_{11}(q) & N_{12}(q) \\ N_{21}(q) & N_{22}(q) \end{bmatrix} & \begin{array}{c} n-r \\ r \end{array} \\ \begin{array}{cc} r & n-r \end{array} & \end{array} \qquad (9)$$

The state variables $\delta\phi, \delta\psi, \delta\dot{\phi}, \delta\dot{\psi}$ consist of a deviation of joint displacement and velocity from the equilibrium point and the input is the generalized force τ of the active joints. (In this case, the number of r and choice of ϕ, ψ is arbitrary, so it is not necessary for ψ to include all the passive joints.)

3.2 Controllability

When the controllability matrix V is calculated in order to investigate the controllability of system (9),

$$V = [B, AB, A^2B, \cdots, A^{n-1}B] = \begin{bmatrix} 0 & N_{11} & 0 \cdots\cdots & 0 \\ 0 & N_{21} & 0 \cdots\cdots & 0 \\ N_{11} & 0 & 0 \cdots\cdots & 0 \\ N_{21} & 0 & 0 \cdots\cdots & 0 \end{bmatrix} \tag{10}$$

Since rank $V=2r<2n$, system (9) is uncontrollable. In other words, when the displacements and velocities of n joints are simultaneously controlled with the generalized force of r (<n) active joints, there exists a domain where the manipulator cannot reach by any input even in the neighborhood of the equilibrium point.

3.3 Output Controllability

Here, output of the system (9) is taken as

$$y = Cx \tag{11}$$

$$y = \begin{bmatrix} \delta\psi \\ \delta\dot{\psi} \end{bmatrix} \qquad C = \begin{bmatrix} 0 & I_r & 0 & 0 \\ 0 & 0 & 0 & I_r \end{bmatrix}$$

The displacement and velocity of r joints is output. If the system is output-controllable, the existence of input τ which transfers output $\delta\psi$, $\delta\dot{\psi}$ from an arbitrary value to zero is guaranteed. The complete condition for output-controllability of system (9) and (11) is that the rank of matrix $N = [CB, CAB, CA^2B, \cdots, CA^{n-1}B]$ equals 2r.

$$N = \begin{bmatrix} 0 & N_{21} & 0 \cdots\cdots & 0 \\ N_{21} & 0 & 0 \cdots\cdots & 0 \end{bmatrix} \tag{12}$$

From (12), the condition of output-controllability is equivalent to non-singularity of N_{21}. Hence if N_{21} is non-singular, it is possible to set ψ and $\dot{\psi}$ at desired values and to perform local positioning in the neighborhood of the equilibrium point.

If the number of the active joints $r \geqq n-r$, we can select the r elements of ψ so as to include the (n-r) passive joints. In that case, all the passive joints can be positioned with the generalized force of the active joints. On the other hand, the displacement and velocity of joints greater than r is not output-controllable irrespective of output matrix C since rank of N is less than or equal to 2r.

Furthermore, it can be proved that this condition agrees with the condition that M_{21} is non-singular, under which (4b) has a solution. This means that if and only if the acceleration of r joints, including (n-r) passive joints, can be arbitrarily determined by the generalized force of r active joints without linear approximation, systems (9) and (11) are output-controllable.

4. CONTROL ALGORITHM

In controlling the position of a manipulator, several control algorithms can be devised according to the way in which the two control modes are combined. As an example, point to point (PTP) control is considered here.

Since the passive joints are controlled with the brakes released (OFF) and the active joints are controlled with the brakes engaged (ON), the control mode should be changed at least one time before all the joints of the manipulator are set at a desired position. It is considered difficult to control both the passive and active joints simultaneously to reach the desired position precisely while the passive joints are free. In the simplest control algorithm sequence, therefore, first the passive joints are positioned while the passive joints are free, then the remaining active joints are positioned while the passive joints are fixed.

PTP control is possible with this sequence alone, but to make it even easier, a brakes-ON period is added before brakes-OFF period to provide kinetic energy to the link mechanism by the active joints so that the passive joints can move easily and unnecessary motion of the active joints is reduced. Consequently, the period of positioning is divided into the following three phases.

Phase I : ψ joints are fixed ($T_0 \leqq t < T_1$, brakes ON)
Phase II : ψ joints are free ($T_1 \leqq t < T_2$, brakes OFF)
Phase III : ψ joints are fixed ($T_2 \leqq t < T_3$, brakes ON)

At phase II, r joints, including passive joints, are controlled along a desired trajectory, and in phases I and III, the remaining (n-r) active joints are controlled from the initial position to the final position.

First of all, the trajectory and the actuator torque in phase II are calculated off-line as follows (Fig. 1).

① Desired displacement, velocity and acceleration $\psi_d(t)$, $\dot{\psi}_d(t)$, $\ddot{\psi}_d(t)$ $(T_1 \leqq t \leqq T_2)$ of r joints, including all the passive joints, are prescribed.
(However, $\dot{\psi}_d(T_1) = \dot{\psi}_d(T_2) = 0$ is a boundary condition because at the moment of brakes ON/OFF, the passive joints must be at rest.)
② The initial displacement and velocity of the remaining (n-r) joints $\phi(T_1)$, $\dot{\phi}(T_1)$ are assigned.
③ (5) and (6) are calculated to determine $\tau(T_1)$, $\ddot{\phi}(T_1)$.
④ $\phi(T_1+\Delta T)$ and $\dot{\phi}(T_1+\Delta T)$ are determined by numerical integration of $\ddot{\phi}(t)$. However, ΔT is the sampling interval.
⑤ {③ calculation of (5), (6) → ④ numerical integration} is repeated and the torque τ, acceleration $\ddot{q}$, velocity $\dot{q}$ and displacement q are determined for the whole of phase II.

The results of these off-line calculations are used for feed-forward compensation. Moreover, the boundary values of ϕ, $\dot{\phi}$ between the phases I, III and II are determined using this result.

When control is executed in real time, the following feedback control is applied.

$$\ddot{\psi}_d' = \ddot{\psi}_d + K_v(\dot{\psi}_d - \dot{\psi}) + K_p(\psi_d - \psi) \tag{13}$$

(K_v, K_p : diagonal gain matrix)

Here ψ_d, $\dot{\psi}_d$, and $\ddot{\psi}_d$ are the desired values of displacement, velocity and acceleration given in ① and ψ, $\dot{\psi}$ are the measured values of displacement and velocity. Acceleration $\ddot{\psi}_d'$ obtained in (13) is substituted in $\ddot{\psi}_d$ of (6) and the torque τ is determined. In this case, real values for ϕ and $\dot{\phi}$ can be obtained by measurement without numerical integration. From (4a),(5),(6) and (13),

$$(\ddot{\psi}_d - \ddot{\psi}) + K_v(\dot{\psi}_d - \dot{\psi}) + K_p(\psi_d - \psi) = 0 \tag{14}$$

Therefore, $\psi_d - \psi$ is guaranteed to converge to zero if K_p and K_v are chosen adequately.

In phases I and III, control of ϕ is similar to an ordinary manipulator. In phase I, ϕ is controlled from the initial position $\phi(T_0)$ to $\phi(T_1)$ and $\dot{\phi}$ is controlled from $\dot{\phi}(T_0)$ = zero to $\dot{\phi}(T_1)$. In phase III, ϕ is controlled from $\phi(T_2)$ to final position $\phi(T_3)$ and $\dot{\phi}$ is controlled from $\dot{\phi}(T_2)$ to $\dot{\phi}(T_3)$ = zero.

With the algorithm mentioned above, position of a manipulator can be controlled between any two points.

5. SIMULATION

In order to confirm the feasibility of the proposed method, control of a two degree of freedom manipulator (Fig. 2) is considered. The torque of the active joint required for positioning, positioning time, etc. are evaluated. Specifically, equations (5) and (6) were expressed as follows.

$$\ddot{\phi} = -\frac{M_{22}\ddot{\psi} + D_{211}(\psi)\dot{\phi}^2}{M_{21}(\psi)} \tag{15}$$

$$\tau = (M_{12}(\psi) - \frac{M_{11}(\psi)M_{22}}{M_{21}(\psi)})\ddot{\psi} + D_{112}(\psi)\dot{\phi}\dot{\psi} + D_{122}(\psi)\dot{\psi}^2 - \frac{M_{11}(\psi)D_{211}(\psi)}{M_{21}(\psi)}\dot{\phi}^2 \tag{16}$$

The trajectory of ψ in phase II is assigned arbitrarily. The acceleration/ deceleration curve is chosen to follow the sinusoidal function in order to reduce the required torque of the actuator. The maximum value of $|\ddot{\psi}|$ are assigned so that the torque of the actuator is limited to a certain value. T_2-T_1 is obtained from the values of $\psi(T_1)$ and $\psi(T_2)$, and the trajectory of ψ is assigned.

The initial values of ϕ and $\dot{\phi}$ ($\phi(T_1)$, $\dot{\phi}(T_1)$) can be arbitrarily selected. The trajectory of ϕ is determined by the algorithms ② thru ⑤ in Chapter 4, and thereafter $\phi(T_2)$ and $\dot{\phi}(T_2)$ is calculated. The acceleration/ deceleration of ϕ along I and III also follows the sinusoidal function.

An example of PTP control is illustrated with a stick diagram (Fig.3). Since (15) and (16) have solutions when $M_{21}\neq 0$, control of the manipulator is possible within the domain of $-2.30<\psi<2.30$. Within the following domain the values of $\psi(T_0)$, $\psi(T_3)$ are varied by $\pi/6$ steps, and the maximum positioning time, maximum torque and so on are evaluated (Table 1). In this case, the maximum value of $|\ddot{\phi}|$ in I, III and that of $|\ddot{\psi}|$ in II are 4π rad/s^2.

$$\phi(T_0)=0,\ -\pi/2\leqq\psi(T_0)\leqq\pi/2,\ 0\leqq\phi(T_3)\leqq\pi/2,\ -\pi/2\leqq\psi(T_3)\leqq\pi/2$$

Within this domain, positioning between any two points is completed within 1.8s, and the performance required of the actuator and the brake are satisfied with products that can be easily obtained.

6. CONCLUSION

In this paper, a method has been proposed for controlling the position of a manipulator which has passive joints comprised of only holding brakes and sensors by using dynamic coupling. It also describes the basic principles and the conditions which make control possible. The feasibility of the method is demonstrated by the simulation of a manipulator with two degrees of freedom. Fundamental experiments are now being carried out with an experimental manipulator.

Since the proposed method depends on a dynamic model of a manipulator, this method is more effective when applied in combination with dynamic modelling and identification methods.

We judged the controllability of a manipulator by the non-singularity of an inertia matrix only qualitatively. However, it is preferable to represent the ability of control with quantitative criteria like output-controllability gain[8].

When some joint actuators of a manipulator are exchanged for holding brakes with the proposed method, weight, energy consumption and cost of the manipulator are expected to be reduced. We can take advantage of these merits by applying them to simple assembly robots, manipulator redundancy control, etc. Space applications (e.g. space manipulators, expansion of space structures) are also effective because there is no influence of gravity.

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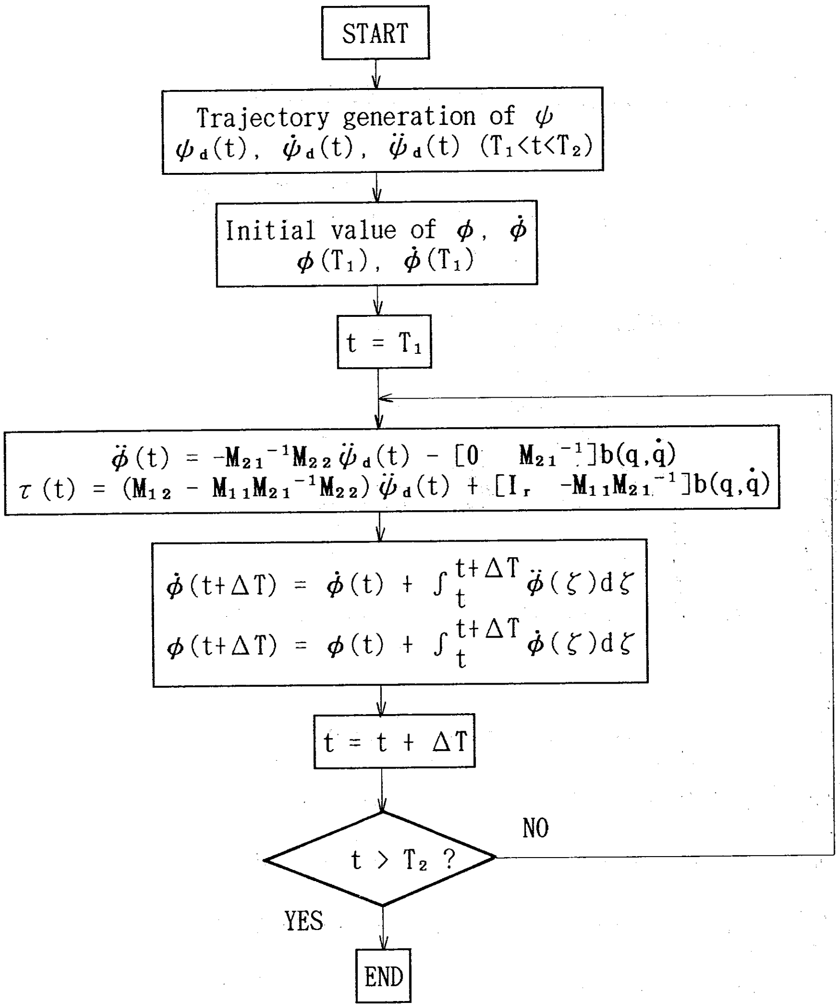

Fig.1 Control Algorithm in Brake-OFF Period

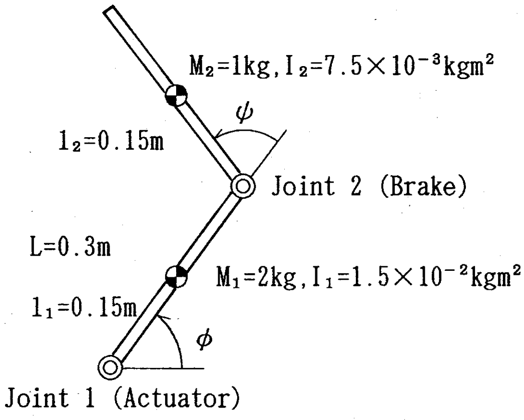

Fig.2 Two Degrees of Freedom Manipulator

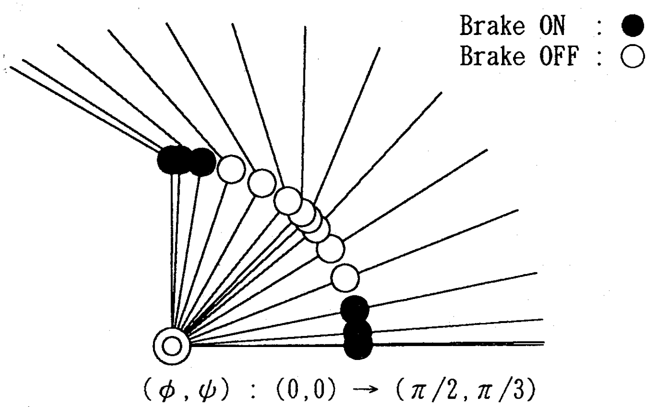

Fig.3 Simulated Motion of the Manipulator

Table 1 Simulation Result

Positioning Time	$T_3 \leqq 1.78$s
Actuator Torque	$\lvert\tau_\phi\rvert \leqq 2.3$N·m
Actuator Velocity	$\lvert\dot{\phi}\rvert \leqq 2.9$rad/s
Actuator Acceleration	$\lvert\ddot{\phi}\rvert \leqq 12.6$rad/s^2
Brake Torque	$\lvert\tau_\psi\rvert \leqq 0.96$N·m